

Chapter 7.1 Systems of Linear Eq. in 2 Variables

① Decide whether an ordered pair is a solution of a linear system

Ex)
$$\begin{cases} 2x - 3y = -4 \\ 2x + y = 4 \end{cases}$$

a.) (1, 2)

$$\begin{aligned} 2(1) - 3(2) &= -4 \\ 2 - 6 &= -4 \\ -4 &= -4 \text{ True} \end{aligned}$$

$$\begin{aligned} 2(1) + 2 &= 4 \\ 2 + 2 &= 4 \text{ True} \\ 4 &= 4 \end{aligned}$$

∴ Solution

b.) (7, 6)

$$\begin{aligned} 2(7) - 3(6) &= -4 \\ 14 - 18 &= -4 \\ -4 &= -4 \text{ True} \end{aligned}$$

$$\begin{aligned} 2(7) + 6 &= 4 \\ 14 + 6 &= 4 \text{ False} \\ 20 &= 4 \end{aligned}$$

Not solution

② Solve linear systems by substitution

Ex)
$$\begin{cases} 3x + 2y = 4 \\ 2x + y = 1 \end{cases}$$

$$\begin{aligned} y = -2x + 1 &\Rightarrow 3x + 2(-2x + 1) = 4 \\ 3x - 4x + 2 &= 4 \\ -x + 2 &= 4 \\ x &= -2 \end{aligned}$$

$$\begin{aligned} y &= -2(-2) + 1 \\ y &= 4 + 1 \\ y &= 5 \end{aligned}$$

plug in
$$\boxed{(-2, 5)}$$

③ Solve linear systems by addition

Ex) Solve w/ addition

$$\begin{cases} 4x + 5y = 3 \\ 2x - 3y = 7 \cdot -2 \end{cases}$$

Ex cont'd

$$\begin{array}{r}
 4x + 5y = 3 \\
 + \quad -4x + 6y = -14 \\
 \hline
 0 + 11y = -11 \\
 \frac{11y}{11} = \frac{-11}{11} \\
 y = -1
 \end{array}$$

$$\begin{array}{r}
 4x + 5(-1) = 3 \\
 4x - 5 = 3 \\
 4x = 8 \\
 x = 2
 \end{array}$$

↑ plug in

$$\boxed{(2, -1)}$$

Ex

$$\begin{cases}
 2x = 9 + 3y \\
 4y = 8 - 3x
 \end{cases}$$

$$\begin{cases}
 2x - 3y = 9 \cdot 4 \\
 3x + 4y = 8 \cdot 3
 \end{cases}$$

$$\begin{array}{r}
 8x - 12y = 36 \\
 + \quad 9x + 12y = 24 \\
 \hline
 17x = 60 \\
 x = \frac{60}{17}
 \end{array}$$

Plug in

$$4y = 8 - 3\left(\frac{60}{17}\right)$$

$$4y = 8 - \frac{180}{17}$$

$$4y = \frac{136}{17} - \frac{180}{17}$$

$$\frac{4y}{4} = -\frac{44}{17} \cdot \frac{1}{4}$$

$$y = -\frac{11}{17}$$

$$\boxed{\left(\frac{60}{17}, -\frac{11}{17}\right)}$$

Note:

When everything cancels and is $0 = 14$ No Solution
 or
 $10 = 10$ Infinite many Solutions

- ⑤ Solve problems using systems of linear equations. C 7.1

Ex | Pg 738 in book CP 7

x = liters of 18% acid

y = liters of 45% acid

$$x + y = 12$$

$$0.18x + 0.45y = 0.36(12)$$

$$y = 12 - x$$

$$0.18x + 0.45(12 - x) = 4.32$$

$$0.18x + 5.4 - 0.45x = 4.32$$

$$-0.27x + 5.4 = 4.32$$

$$\frac{-0.27x}{-0.27} = \frac{-1.08}{-0.27}$$

$$x = 4$$

∴ 4 liters of 18% acid and 8 liters of 45% acid

Ex | pg 739 in book CP 8

x = velocity of boat

y = velocity of current

Velocity	Time	Distance
$x + y$	2	$2(x + y)$
$x - y$	3	$3(x - y)$

$$2(x + y) = 84$$

$$3(x - y) = 84$$

$$x + y = 42$$

$$+ x - y = 29$$

$$\hline 2x = 71$$

$$\underline{x = 35.5}$$

$$x + y = 42$$

$$35.5 + y = 42$$

$$\underline{y = 6.5}$$

velocity of boat

35.5 mph

current

6.5 mph

Pg 3

Chapter 7.2 Systems of Linear Eq in 3 variables

① Verify the solution of a system of linear equations in 3 variables

Ex) Show that the ordered triple $(-1, -4, 5)$ is a solution

$$\begin{cases} x - 2y + 3z = 22 \\ 2x - 3y - z = 5 \\ 3x + y - 5z = -32 \end{cases}$$

1st eq

$$\begin{aligned} -1 - 2(-4) + 3(5) &= 22 \\ -1 + 8 + 15 &= 22 \\ 22 &= 22 \\ \text{True} \end{aligned}$$

2nd eq

$$\begin{aligned} 2(-1) - 3(-4) - 5 &= 5 \\ -2 + 12 - 5 &= 5 \\ 5 &= 5 \\ \text{True} \end{aligned}$$

3rd eq

$$\begin{aligned} 3(-1) + (-4) - 5(5) &= -32 \\ -3 - 4 - 25 &= -32 \\ -32 &= -32 \\ \text{True} \end{aligned}$$

② Solve systems of linear equations in 3 variables.

Ex) $\begin{cases} \text{Eq 1} & x + 4y - z = 20 \\ \text{Eq 2} & 3x + 2y + z = 8 \\ \text{Eq 3} & 2x - 3y + 2z = -16 \end{cases}$

$\begin{aligned} \text{Eq 4} & 4x + 6y = 28 \\ \text{Eq 5} & -4x - 5y = -32 \\ \hline & y = -4 \end{aligned}$

$\begin{aligned} \text{Eq 1} & x + 4y - z = 20 \\ \text{Eq 2} & 3x + 2y + z = 8 \\ \text{Eq 4} & 4x + 6y = 28 \end{aligned}$

$\begin{aligned} \text{Eq 4} & 4x + 6(-4) = 28 \\ 4x - 24 &= 28 \\ 4x &= 52 \\ x &= 13 \end{aligned}$

$(13, -4, -23)$

$\text{Eq 2 (times -2)} \quad -6x - 2y - 2z = -16$

$\text{Eq 3} \quad 2x - 3y + 2z = -16$

$\text{Eq 5} \quad -4x - 5y = -32$

$\begin{aligned} \text{Eq 1} & 13 + 4(-4) - z = 20 \\ 13 - 16 - z &= 20 \\ -3 - z &= 20 \\ -z &= 23 \\ z &= -23 \end{aligned}$

~~13, -4, -23~~
Pg 1

[Ex] Solve the system

$$\begin{cases} 2y - z = 7 \\ x + 2y + z = 17 \\ 2x - 3y + 2z = -1 \end{cases}$$

$$\begin{array}{r} -2x - 4y - 2z = -34 \\ 2x - 3y + 2z = -1 \\ \hline \end{array}$$

$$-7y = -35$$

$$y = 5$$

$$2(5) - z = 7$$

$$10 - z = 7$$

$$-z = -3$$

$$z = 3$$

$$x + 2(5) + 3 = 17$$

$$x + 13 = 17$$

$$x = 4$$

$$\boxed{(4, 5, 3)}$$

③ ~~Solve problems~~ using systems in 3 variables

[Ex]

7.3 Partial Fractions

Ex: $\frac{7}{12} = \frac{1}{3} + \frac{1}{4}$ ~~$\frac{1}{3} + \frac{1}{4}$~~ ~~$\frac{1}{3} + \frac{1}{4}$~~

The process of starting with the simplified answer and taking it back apart

- "Decomposing" the final expression into its initial polynomial fractions.

EX: Going from $\frac{x+14}{x^2-2x-8}$ back to $\frac{3}{x-4} - \frac{2}{x+2}$

- In calculus, you cannot integrate the final result. You have to integrate each part separately.

$$\frac{x+14}{x^2-2x-8} = \frac{A}{x-4} + \frac{B}{x+2}$$

- Depends on the factors of the denominator:
1. Denominator is the product of distinct linear factors
 2. Denominator is the product of linear factors, some of which are repeated
 3. Denominator has prime quadratic factors, none of which are repeated
 4. Denominator has a repeated prime quadratic factor

$x+14 = A(x+2) + B(x-4)$
 if $x = -2$ $-2+14 = A(0) + B(-6)$
 $12 = -6B$
 $B = -2$
 if $x = 4$
 $4+14 = A(4+2)$
 $\frac{18}{6} = \frac{6A}{6}$
 $A = 3$

*The numerator should always be one degree less than the factor of the denominator

$$6) \frac{x^2+x+2}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

$$0x^2+x+2 = A(x-1)^2 + Bx(x-1) + Cx$$

if $x=1$ $1+2 = A(0)^2 + B(1)(0) + C$
 $C = 3$

if $x=0$ $0+2 = A(-1)^2 + B(0) + C(0)$ Plug in random #s for x .
 $A = 2$

quad: $0x^2 = Ax^2 + Bx$

$$0 = A + B$$

$$0 = 2 + B$$

$$B = -2$$

or

$$\frac{2}{x} + \frac{-2}{x-1} + \frac{3}{(x-1)^2}$$

$$7) \frac{8x^2+12x-20}{(x+3)(x^2+x+2)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+x+2}$$

$$8x^2+12x-20 = A(x^2+x+2) + (Bx+C)(x+3)$$

if $x=-3$ $8(-3)^2+12(-3)-20 = A(-3)^2-3+2 + (Bx+C)(0)$
 $72-36-20 = A(9-1)$

$$16 = A(8)$$

$$16 = A(8)$$

$$A = 2$$

quad: $8x^2 = Ax^2 + Bx^2$
 $8x = 2 + B$ $B = 6$

constant: $-20 = 2A + 3C$

$$-20 = 2(2) + 3C$$

$$\frac{-24}{3} = \frac{3C}{3}$$

$$C = -8$$

$$8) \frac{2x^3+x+3}{(x^2+1)^2}$$

$$2x^3+x+3 = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{(x^2+1)^2}$$

$$2x^3+x+3 = (Ax+B)(x^2+1) + Cx+D$$

$$2x^3 = Ax^3$$

$$A = 2$$

$$1x = Ax + Cx \quad 3 = B + D$$

$$1 = 2 + C$$

$$3 = 0 + D$$

$$C = -1$$

$$D = 3$$

$$0x^2 = Bx^2$$

$$B = 0$$

$$= \frac{2x}{x^2+1} + \frac{-x+3}{(x^2+1)^2}$$

Write the form of the partial fraction decomposition of the rational expression. It is not necessary to solve for the constants.

$$1) \frac{5x+7}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$$

$$2) \frac{3x+16}{(x+1)(x-2)^2} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$$3) \frac{5x^2-6x+7}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$$

$$4) \frac{7x^2-9x+3}{(x^2+7)^2} = \frac{Ax+B}{x^2+7} + \frac{Cx+D}{(x^2+7)^2}$$

Find the partial fraction decomposition:

$$5) \frac{5x-1}{(x+4)(x-3)} = \frac{A}{x+4} + \frac{B}{x-3}$$

$$5x-1 = A(x-3) + B(x+4)$$

if $x=3$ $5(3)-1 = A(0) + B(7)$

$$14 = 7B$$

$$B = 2$$

$$\boxed{\frac{3}{x+4} + \frac{2}{x-3}}$$

if $x=-4$ $5(-4)-1 = A(-7) + B(0)$

$$-21 = -7A$$

$$A = 3$$

$$6) \frac{x^2+x+2}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

$$0x^2+x+2 = A(x-1)^2 + Bx(x-1) + Cx$$

if $x=1$ $1+2 = A(0)^2 + B(1)(0) + C$
 $C = 3$

if $x=0$ $0+2 = A(-1)^2 + B(0) + C(0)$ Plug in random #s for x.
 $A = 2$

quad: $0x^2 = Ax^2 + Bx^2$

$$0 = A + B$$

$$0 = 2 + B$$

$$B = -2$$

or

$$\frac{2}{x} + \frac{-2}{x-1} + \frac{3}{(x-1)^2}$$

$$7) \frac{8x^2+12x-20}{(x+3)(x^2+x+2)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+x+2}$$

$$8x^2+12x-20 = A(x^2+x+2) + (Bx+C)(x+3)$$

if $x=-3$ $8(-3)^2+12(-3)-20 = A((-3)^2-3+2) + (Bx+C)(0)$
 $72-36-20 = A(9-1)$

$$16 = A(8)$$

$$A = 2$$

quad: $8x^2 = Ax^2 + Bx^2$
 $8 = 2 + B$ $B = 6$

constant: $-20 = 2A + 3C$

$$-20 = 2(2) + 3C$$

$$\frac{-24}{3} = \frac{3C}{3}$$

$$C = -8$$

$$8) \frac{2x^3+x+3}{(x^2+1)^2}$$

$$= \frac{2}{x+3} + \frac{6x-8}{x^2+x+2}$$

$$2x^3+x+3 = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{(x^2+1)^2}$$

$$2x^3+x+3 = (Ax+B)(x^2+1) + Cx+D$$

$$2x^3 = Ax^3$$

$$A = 2$$

$$1x = Ax + Cx \quad 3 = B + D$$

$$1 = 2 + C$$

$$3 = 0 + D$$

$$C = -1$$

$$D = 3$$

$$0x^2 = Bx^2$$

$$B = 0$$

$$= \frac{2x}{x^2+1} + \frac{-x+3}{(x^2+1)^2}$$

$$9) \frac{x+14}{(x-4)(x+2)} = \frac{A}{x-4} + \frac{B}{x+2}$$

$$x+14 = A(x+2) + B(x-4)$$

$$\text{if } x = -2 \quad -2+14 = A(0) + B(-6)$$

$$12 = -6B$$

$$B = -2$$

$$\text{if } x = 4 \quad 4+14 = A(6) + B(0)$$

$$18 = 6A \quad A = 3$$

$$\boxed{\frac{3}{x-4} - \frac{2}{x+2}}$$

$$10) \frac{x-18}{x(x+3)^2} = \frac{A}{x} + \frac{B}{x+3} + \frac{C}{(x+3)^2}$$

$$0x^2 + x - 18 = A(x+3)^2 + Bx(x+3) + Cx$$

$$\text{if } x = -3 \quad -3-18 = A(0)^2 + B(-3)(0) + C(-3)$$

$$-21 = C \cdot (-3)$$

$$C = 7$$

$$\text{if } x = 0$$

$$0-18 = A(3)^2 + B(0) + C(0)$$

$$-18 = 9A$$

$$A = -2$$

quad: $0x^2 = Ax^2 + Bx^2$

$$0 = -2 + B$$

$$B = 2$$

$$\frac{-2}{x} + \frac{2}{x+3} + \frac{7}{(x+3)^2}$$

$$11) \frac{3x^2 + 17x + 14}{(x-2)(x^2 + 2x + 4)} = \frac{A}{x-2} + \frac{Bx+C}{x^2 + 2x + 4}$$

$$3x^2 + 17x + 14 = A(x^2 + 2x + 4) + (Bx + C)(x-2)$$

$$\text{if } x = 2 \quad 3(2)^2 + 17(2) + 14 = A(2^2 + 2(2) + 4) + (Bx + C)(0)$$

$$12 + 34 + 14 = 12A$$

$$60 = 12A$$

$$A = 5$$

$$\text{Constants: } 14 = 4A - 2C$$

$$14 = 4(5) - 2C$$

$$-6 = -2C$$

$$C = 3$$

quad. $3x^2 = Ax^2 + Bx^2$

$$3 = 5 + B$$

$$B = -2$$

$$\boxed{\frac{5}{x-2} + \frac{-2x+3}{x^2+2x+4}}$$

Chapter 7.4 Systems of Nonlinear Eq in 2 Variables

① Recognize systems of nonlinear equations in 2 variables

$$x^2 = 2y + 10 \text{ or } y = x^2 + 3 \text{ or } x^2 + y^2 = 9$$

② Solve nonlinear systems by substitution

Ex] $x^2 = y - 1$
 $4x - y = -1$

$$y = x^2 + 1$$
$$4x - (x^2 + 1) = -1$$
$$4x - x^2 - 1 = -1$$
$$0 = x^2 - 4x$$
$$0 = x(x - 4)$$
$$x = 0, 4$$

if $x = 0$
 $y = 0^2 + 1 = 1$

if $x = 4$
 $y = 4^2 + 1 = 17$

$$\{(0, 1), (4, 17)\}$$

Ex] Solve by substitution

$$\begin{cases} x + 2y = 0 \\ (x-1)^2 + (y-1)^2 = 5 \end{cases}$$

$$x = -2y$$

$$(-2y-1)^2 + (y-1)^2 = 5$$

$$4y^2 + 2y + 2y + 1 + y^2 - 2y + 1 = 5$$

$$5y^2 + 2y - 3 = 0$$

$$(5y-3)(y+1) = 0$$

$$y = 3/5, -1$$

if $y = 3/5$
 $x = -2(3/5) = -6/5$

if $y = -1$
 $x = -2(-1) = 2$

$$\{(-6/5, 3/5), (2, -1)\}$$

③ Solve nonlinear systems by addition

C 7.4

Ex)

$$\begin{cases} 3x^2 + 2y^2 = 35 \cdot -3 \\ 4x^2 + 3y^2 = 48 \cdot 2 \end{cases}$$

$$-9x^2 - 6y^2 = -105$$

$$8x^2 + 6y^2 = 96$$

$$-x^2 = -9$$

$$\sqrt{x^2} = \sqrt{9}$$

$$x = \pm 3$$

if $x = 3$

$$3(3)^2 + 2y^2 = 35$$

$$3(9) + 2y^2 = 35$$

$$27 + 2y^2 = 35$$

$$\frac{2y^2 = 8}{2}$$

$$\sqrt{y^2} = \sqrt{4}$$

$$y = \pm 2$$

Same if $x = -3$

$$\therefore \{(3, 2), (3, -2), (-3, 2), (-3, -2)\}$$

Ex)

$$\begin{cases} y = x^2 + 5 \\ x^2 + y^2 = 25 \end{cases}$$

rearrange

$$\begin{cases} -x^2 + y = 5 \\ x^2 + y^2 = 25 \end{cases}$$

$$y^2 + y = 30$$

$$y^2 + y - 30 = 0$$

$$(y + 6)(y - 5) = 0$$

$$y = -6, 5$$

if $y = -6$

$$x^2 + (-6)^2 = 25$$

$$x^2 + 36 = 25$$

$$\sqrt{x^2} = \sqrt{-11}$$

Not a real #

if $y = 5$

$$x^2 + (5)^2 = 25$$

$$x^2 + 25 = 25$$

$$\sqrt{x^2} = \sqrt{0}$$

$$x = 0$$

$$\{(0, 5)\}$$

④ Solve problems using systems of nonlinear eq. C 7.4

Ex Find the length and width of a rectangle whose perimeter is 20 feet and whose area is 21 ft²

$$\begin{aligned}2x + 2y &= 20 \\ xy &= 21\end{aligned}$$

$$x = \frac{21}{y}$$

$$2\left(\frac{21}{y}\right) + 2y = 20$$

$$y \cdot \frac{42}{y} + 2y = 20 \cdot y$$

$$\frac{42 + 2y^2}{2} = 20y$$

$$21 + y^2 = 10y$$

$$y^2 - 10y + 21 = 0$$

$$(y - 3)(y - 7) = 0$$

$$y = 3, 7$$

$$\underline{\text{if } y = 3}$$

$$\frac{x \cdot 3}{3} = \frac{21}{3}$$

$$x = 7$$

length 7
width 3

$$\underline{\text{if } y = 7}$$

$$x \cdot 7 = 21$$

$$x = 3$$

or
length = 3
width = 7

* Show that a point is a solution. ie (3,8) is a solution to first example.

7.5: Graphs of Linear and Non-Linear Inequalities

Try this with your group.

Graph the solution set for the following system.

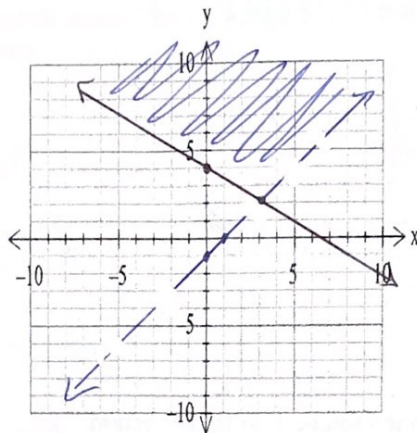
$$\begin{cases} x - y < 1 \\ 2x + 3y \geq 12 \end{cases}$$

$$-y < -x + 1$$

$$y > x - 1$$

$$3y \geq -2x + 12$$

$$y \geq -\frac{2}{3}x + 4$$



Check

$$\begin{aligned} 0 - 0 &< 1 \\ 0 &< 1 \quad \checkmark \end{aligned}$$

$$\begin{aligned} 0 + 0 &\geq 12 \\ 0 &\geq 12 \quad \text{no} \end{aligned}$$

Vertex Form of a Parabola:

$$y = a(x-h)^2 + k$$

Vertex: (h, k)

Standard Form of a Circle:

$$(x-h)^2 + (y-k)^2 = r^2$$

Center: (h, k)

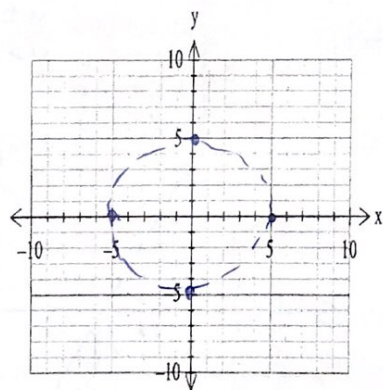
Radius: r

Graph each part of the system, then graph the solution set to the system.

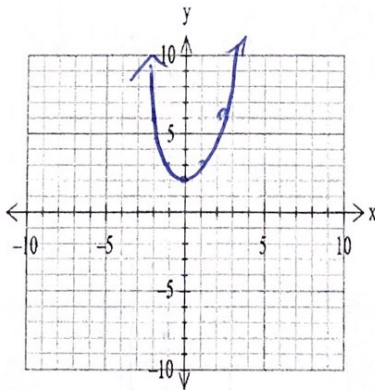
$$\begin{cases} x^2 + y^2 < 25 \\ y \geq x^2 + 2 \end{cases}$$

Plug in (0,0)

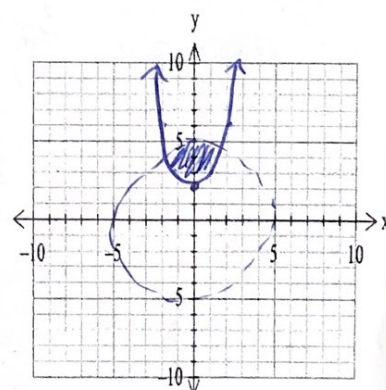
Circle



Parabola



Solution

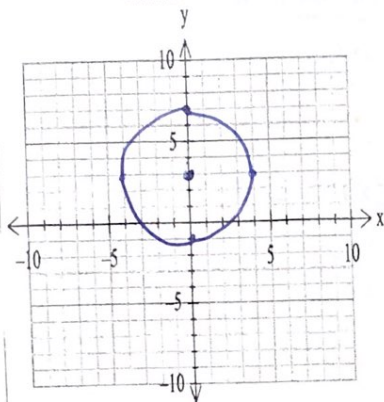


Graph each part of the system, then graph the solution set to the system.

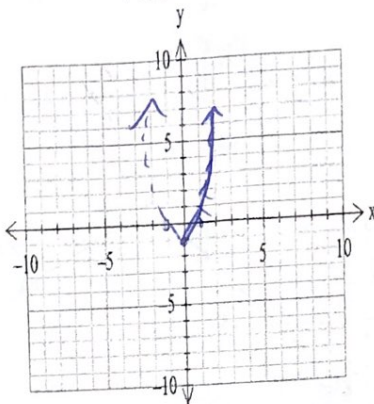
$$\begin{cases} x^2 + (y-3)^2 \leq 16 \\ y < x^2 - 1 \end{cases} \quad \text{center: } (0, 3) \quad r = 4$$

Check
 $0^2 + (-3)^2 \leq 16$
 $9 \leq 16$ ✓
 $0 < -1$ ✓

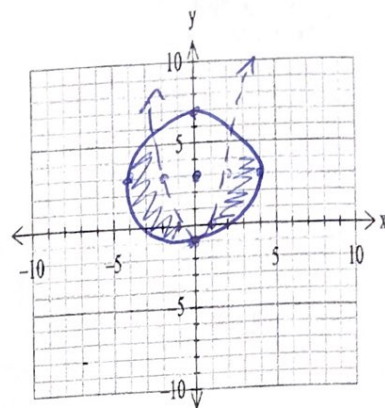
Circle



Parabola



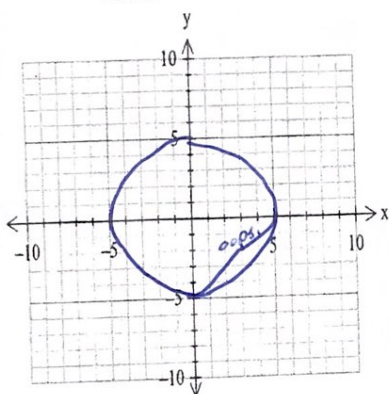
Solution



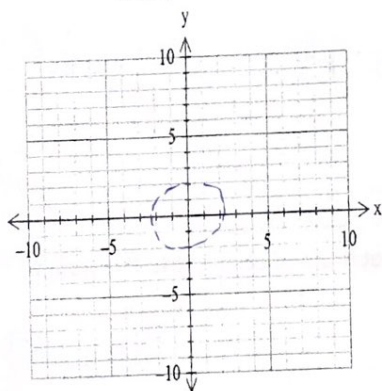
Graph each part of the system, then graph the solution set to the system.

$$\begin{cases} x^2 + y^2 \leq 25 \\ x^2 + y^2 > 4 \end{cases}$$

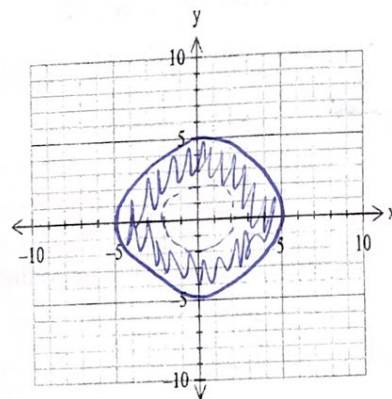
Circle



Circle



Solution



Graph the solution set to each problem below.

1. $\begin{cases} 5x + 3y \leq 15 \\ y < \frac{1}{2}x^2 - 4 \end{cases}$

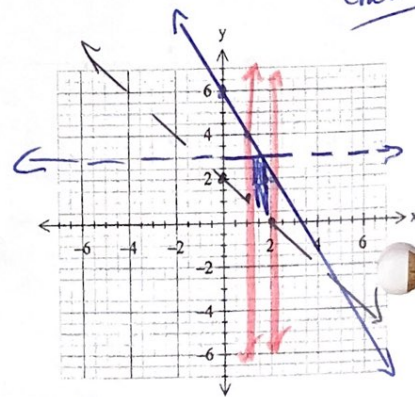
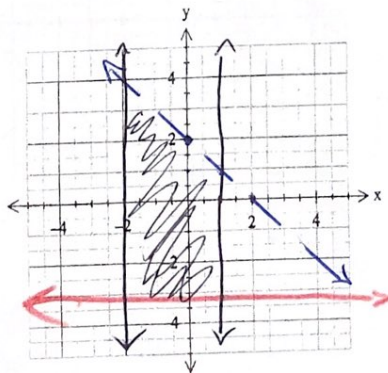
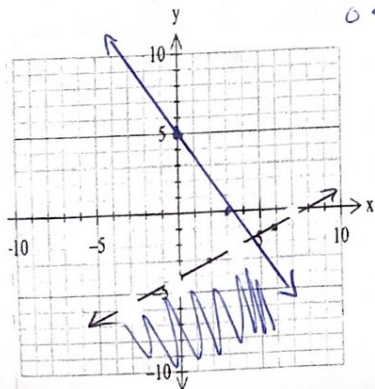
$3y \leq -5x + 15$
 $y \leq -\frac{5}{3}x + 5$

2. $\begin{cases} x + y < 2 \\ -2 \leq x < 1 \\ y > -3 \end{cases}$

$y < -x + 2$

3. $\begin{cases} 2x + y \leq 6 \\ x + y > 2 \\ 1 \leq x \leq 2 \\ y < 3 \end{cases}$

Check
 $0 \leq 15$ ✓
 $0 < -4$ ✓



check

Hawel

7.6 Linear Programming

1) A small airline company in Colorado only flies to two cities and can have up to 10 flights per day. Each flight to San Francisco takes two hours, and each flight to Seattle takes three hours. The company must offer at least 3 flights to San Francisco and 2 flights to Seattle each day. Each flight to San Francisco makes the company \$1000, and each flight to Seattle makes the company \$1200 profit. How many flights to each city should be offered to make the maximum profit?

Let x = Flights to S.F.

Let y = Flights to Sea.

Objective Function: $1000x + 1200y$

Constraints:

$$x + y \leq 10 \quad y \leq -x + 10$$

$$x \geq 3$$

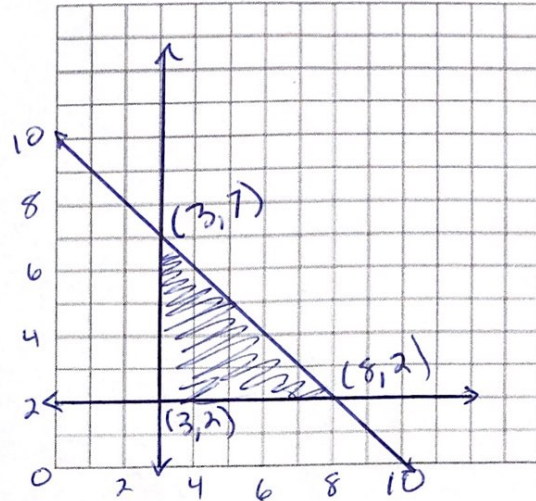
$$y \geq 2$$

Solution: $(3, 2) = \$5400$

$(8, 2) = \$10,400$

$(3, 7) = \$11,400$

\therefore 3 flights to SF & 7 flights to SEA.



2) A store wants to liquidate 200 shirts and 100 pairs of pants from last season. The store has two offers. Offer A is a package of one shirt and one pair of pants for \$30. Offer B is a package of three shirts and one pair of pants for \$50. The store needs to sell at least 20 packages of Offer A and at least 10 packages of Offer B. How many packages of each offer should they sell in order to maximize their money?

Let x = # of packages of Offer A.

Let y = # of packages of Offer B.

Objective Function: $30x + 50y$

Constraints: $x \geq 20$

$$y \geq 10$$

$$\rightarrow \text{shirts} \quad x + 3y \leq 200 \quad y \leq -\frac{1}{3}x + \frac{200}{3}$$

$$\rightarrow \text{pants} \quad x + y \leq 100 \quad y \leq -x + 100$$

Solution:

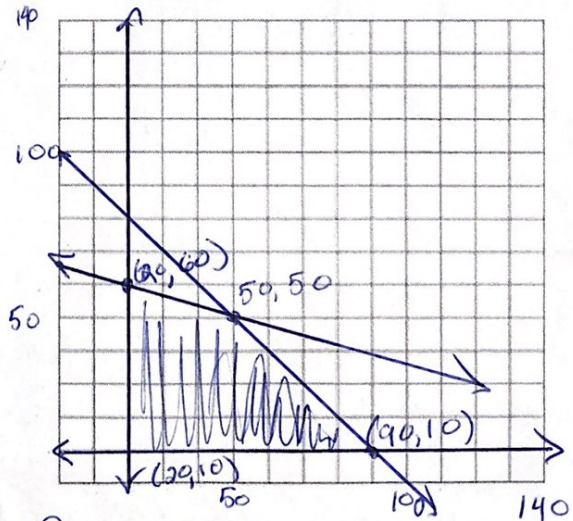
$(20, 10) = \$1100$

$(20, 60) = \$3600$

$(50, 50) = \$4000$

$(90, 10) = \$3200$

\therefore 50 packages of each



3) A gold processor has two sources of gold ore, source A and Source B. In order to keep this plant running, at least three tons of ore must be processed each day. Ore from source A costs \$20 per ton to process, and ore from source B costs \$10 per ton to process. Costs must be kept to less than \$80 per day. Moreover, Federal Regulations require that the amount of ore from source B cannot exceed twice the amount of ore from source A. If ore from source A yields 2 oz. of gold per ton, and ore from source B yields 3 oz. of gold per ton, how many tons of ore from both sources must be processed each day to maximize the amount of gold extracted subject to the above constraints?

Let $x =$ Source A

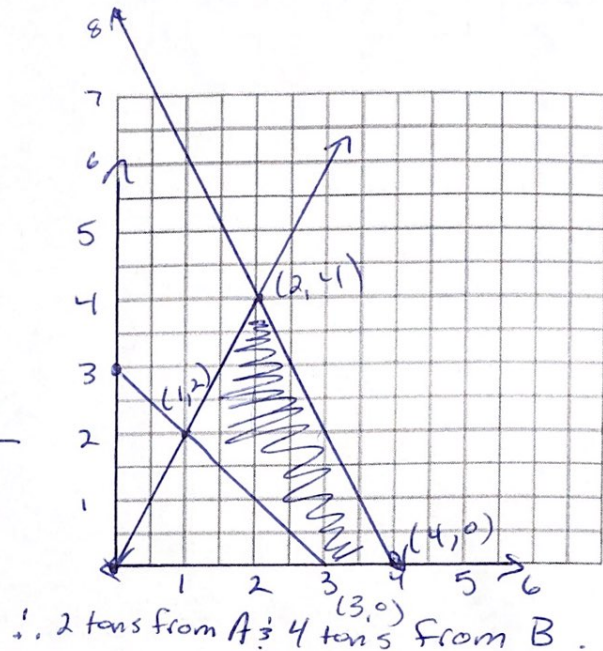
Let $y =$ Source B

Objective Function: $2x + 3y$

Constraints: $x + y \geq 3$ $y \geq -x + 3$
 $20x + 10y \leq 80$ $y \leq -2x + 8$

Solution: $y \leq 2x$ $x \geq 0$ $y \geq 0$

$(3, 0) = 6 \text{ oz}$
 $(4, 0) = 8 \text{ oz}$
 $(1, 2) = 8 \text{ oz}$
 $(2, 4) = 16 \text{ oz}$



4) A company manufactures gold balls and tennis balls. Golf balls are made on Machine A for 10 minutes and Machine B for 6 minutes. Tennis balls need 5 minutes of Machine A, 8 minutes on Machine B, and 4 minutes on Machine C. The company must make at least 100 golf balls and 200 tennis balls per week. Machine A is available for 6500 minutes per week. Machine B is available for 6400 minutes per week, and machine C is available for 2300 minutes per week. The profit for one golf ball is \$1.25, and each tennis ball earns \$1.35 in profit. Find the maximum weekly profit.

Let $x =$ # of golf balls

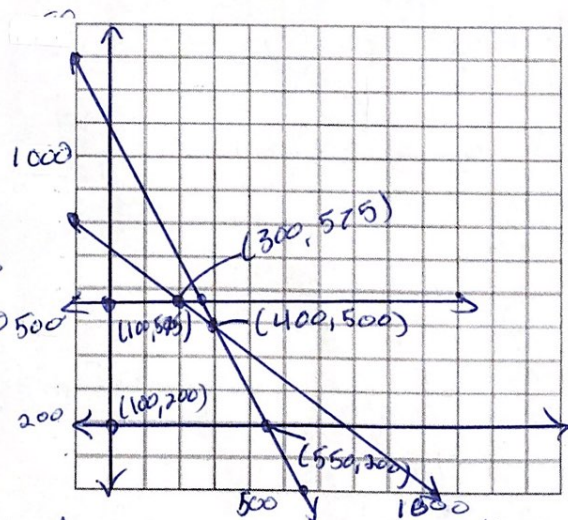
Let $y =$ # of tennis balls

Objective Function: $1.25x + 1.35y$

Constraints: $10x + 5y \leq 6500$ $y \leq -2x + 1300$
 $6x + 8y \leq 6400$ $y \leq -\frac{3}{4}x + 800$
 $4y \leq 2300$ $y \leq 575$

Solution: $x \geq 100$ $y \geq 200$

$(100, 200) = 395$
 $(550, 200) = 957.5$
 $(400, 500) = 1175$
 $(300, 575) = 1151.25$
 $(100, 575) = 901.25$



$\therefore 400$ golf balls & 500 tennis balls = \$1175 profit.